Lecture 10
14.1 Functions of several variables

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Things to note

Exams will be passed back at end of class.

Office hours canceled today and tomorrow.

Quiz on Friday will cover today's material.

HW04 will be posted by 11:30.

Chapter 14: Partial derivatives

Chapter 14 is all about re-doing Calculus 1 with multivariable functions.

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Chapter 12
Vectors
Products
Planes
Lines

Chapter 13
Vector Functions

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Chapter 12
Vectors
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Vector Functions

Chapter 14
Limits
(Partial) Derivatives
The Gradient
(Directional) Derivatives
Tangent planes
Min/Max

14.1 Functions of several variables

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Example

$f(x, y) = x^2 + y^2$, $z = \sin(x + y)$, $g(x, y) = e^y - 5x$

Domains

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We have $x - 1 \neq 0$, so $x \neq 1$. Thus the domain is $(-\infty, 1) \cup (1, \infty)$, or $\{x \in \mathbb{R} | x \neq 1\}$.

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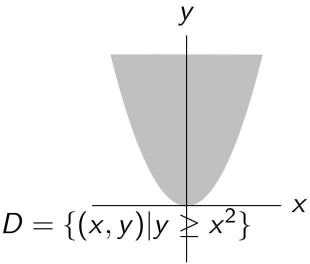
The defining restriction for $f(x, y)$ is

$$y - x^2 \geq 0.$$

This is visualized as



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Example

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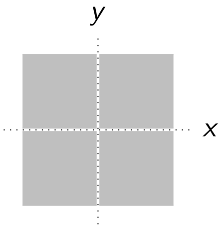
The defining inequality is

$$xy \neq 0 \Rightarrow x \neq 0 \text{ and } y \neq 0.$$

So all the points not on the x- or y-axes.



$D = \{(x, y)|xy \neq 0\}$

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The domain is $D = \mathbb{R}^2$.

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Find the range of $f(x,y) = \sqrt{y - x^2}$ *and* $z = \frac{1}{xy}$.
The range of $f(x,y)$ is $\{z \in \mathbb{R} | z \geq 0\}$ or $[0, \infty)$.

The range of z is $(-\infty, 0) \cup (0, \infty)$ or $\{z \in \mathbb{R} | z \neq 0\}$.

Level curves

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Example

Let $f(x, y) = 100 - x^2 - y^2$. Find the level curves for $z = 100, z = 75, z = 51, and z = 0$.

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For $z = 100$, we get $100 = 100 - x^2 - y^2$, or $x^2 + y^2 = 0$. This defines the point $(0, 0, 100)$.

Level curve example

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Let $f(x,y) = 100 - x^2 - y^2$. *Find the level curves for* $z = 100, z = 75, z = 51, and z = 0$.

For $z = 100$, we get $100 = 100 - x^2 - y^2$, or $x^2 + y^2 = 0$. This defines the point $(0, 0, 100)$.

For $z = 75$, we get $75 = 100 - x^2 - y^2$, or $25 = x^2 + y^2$. This defines the circle of radius 5 centered at $(0, 0, 75)$, lying in the plane $z = 75$.

Level curve example

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Let $f(x,y) = 100 - x^2 - y^2$. *Find the level curves for* $z = 100, z = 75, z = 51, and z = 0$.

For $z = 100$, we get $100 = 100 - x^2 - y^2$, or $x^2 + y^2 = 0$. This defines the point $(0,0,100)$.
For $z = 75$, we get $75 = 100 - x^2 - y^2$, or $25 = x^2 + y^2$. This defines the circle of radius 5 centered at $(0,0,75)$, lying in the plane $z = 75$.
For $z = 51$, we get $51 = 100 - x^2 - y^2$, or $49 = x^2 + y^2$. This defines the circle of radius 7 centered at $(0,0,51)$, lying in the plane $z = 51$.

Level curve example

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For $z = 100$, we get $100 = 100 - x^2 - y^2$, or $x^2 + y^2 = 0$. This defines the point $(0, 0, 100)$.
For $z = 75$, we get $75 = 100 - x^2 - y^2$, or $25 = x^2 + y^2$. This defines the circle of radius 5 centered at $(0, 0, 75)$, lying in the plane $z = 75$.
For $z = 51$, we get $51 = 100 - x^2 - y^2$, or $49 = x^2 + y^2$. This defines the circle of radius 7 centered at $(0, 0, 51)$, lying in the plane $z = 51$.
For $z = 0$, we get $0 = 100 - x^2 - y^2$, or $100 = x^2 + y^2$. This defines the circle of radius 10 centered at $(0, 0, 0)$, lying in the plane $z = 0$.

Level curve example

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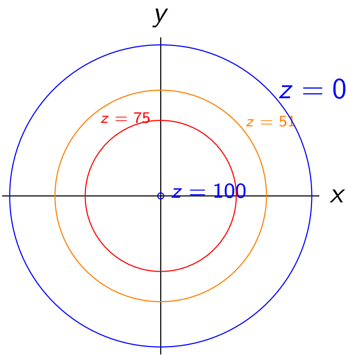

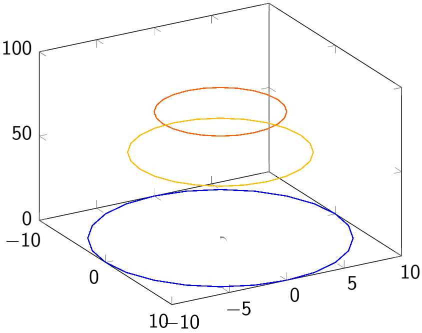

Full picture

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